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The metastable bound state of a pair of two-dimensional spatially separated electrons in antiparallel magnetic fields

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Abstract

We propose a new mechanism for binding of two equally charged carriers in a double-layer system subjected to a magnetic field of a special form. A field configuration for which the magnetic fields in adjacent layers are equal in magnitude and opposite in direction is considered. In such a field an additional integral of motion—the momentum of the pair **P**—arises. For the case where in one layer the carrier is in the zero ($n = 0$) Landau level while in the other layer the carrier is in the first ($n = 1$) Landau level, the dependence of the energy of the pair on its momentum $E(\mathcal{P})$ is found. This dependence turns out to be a nonmonotonic one: a local maximum and a local minimum appear, indicating the emergence of a metastable bound state of two carriers with the same sign of electrical charge.

1. Introduction

During the last ten years the possibility of measuring the effects caused by the interaction of spatially separated carriers in low-dimensional systems has been demonstrated in a number of experiments. Undoubted evidence for such effects was obtained in drag experiments in which a voltage in one conducting layer caused by an electric current in the adjacent layer (separated from the first one by a dielectric layer) was observed. Drag effects have been registered in bilayer systems with conductivity of the same type in both layers (for instance, of the electron type) [1–3] and in layers with conductivities of opposite types (electron type in one layer and hole type in the other one) [4, 5]. In the last case the interaction between spatially separated electrons and holes may result not only in a drag effect but also in an electron–hole pairing. The electron–hole pairs may condense into a specific superfluid state in which a supercurrent in one layer is accompanied by a supercurrent in the adjacent layer and these currents have the same absolute value but the opposite directions [6, 7].

The most favourable conditions for electron–hole pairing are achieved in the case where a strong (quantizing) magnetic field perpendicular to the layers is applied to a bilayer electron–hole [8–10] or electron–electron system [11, 12] (with the total filling factor $\nu_{\mathrm{T}} = 1$ in the last case). The experimental discovery of the superfluidity of the pairs in such systems has already been reported [13, 14].

The possibility of pairing of spatially separated electrons and holes looks quite natural since there are Coulomb attraction forces between an electron and a hole. There is also an unexpected and less obvious phenomenon: in a strong magnetic field the Coulomb repulsion between spatially separated equally charged carriers may result in the formation of metastable bound pairs. Such an effect occurs in a situation where the magnetic fields applied to the first and the second layer of the bilayer electron (or hole) system are antiparallel to each other. This possibility was predicted in our paper [15], where we assume that, together with the antiparallel magnetic fields perpendicular to the layers, electric fields antiparallel to each other and parallel to the conducting layers are applied to the system. The disadvantage of the situation considered in [15] is that the presence of the electric fields may result in an instability of the system with a macroscopic number of the pairs.

In this paper we show that the formation of a metastable bound state of spatially separated electrons (or holes) can emerge without electric fields being applied to the system and formulate the conditions for the appearance of such a bound state.

2. Electron–electron pairs in antiparallel magnetic fields

Let us consider two two-dimensional electron layers with the interlayer distance d embedded in a dielectric matrix with the dielectric constant ε_0. Let the magnetic field in the top layer (layer 1) be $\mathbf{B}_1 = (0, 0, -B)$ and the magnetic field in the bottom layer (layer 2) be $\mathbf{B}_2 = (0, 0, B)$ (the z axis is chosen perpendicular to the layers). A possible way of realizing such a configuration of magnetic fields will be discussed at the end of the paper. We specify the case where there is one electron belonging to the zero Landau level in layer 1 and one electron belonging to the first Landau level in layer 2. In the symmetric gauge the vector potential in layer 1 is equal to $\mathbf{A}_1(\mathbf{r}_1) = (By_1/2, -Bx_1/2, 0)$ and that in layer 2 is $\mathbf{A}_2(\mathbf{r}_2) = (-By_2/2, Bx_2/2, 0)$. The Hamiltonian of a pair of interacting electrons has the form

$$H = H_1 + H_2 + V_{\mathrm{C}}(|\mathbf{r}_1 - \mathbf{r}_2|), \tag{1}$$

where

$$H_1 = \frac{\left(p_{1x} + \frac{eB}{2c}y_1\right)^2}{2m_1} + \frac{\left(p_{1y} - \frac{eB}{2c}x_1\right)^2}{2m_1}, \tag{2}$$

$$H_2 = \frac{\left(p_{2x} - \frac{eB}{2c}y_2\right)^2}{2m_2} + \frac{\left(p_{2y} + \frac{eB}{2c}x_2\right)^2}{2m_2}, \tag{3}$$

$$V_{\mathrm{C}} = \frac{e^2}{\varepsilon_0|\mathbf{r}_1 - \mathbf{r}_2|} = \frac{e^2}{\varepsilon_0\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + d^2}}. \tag{4}$$

Here $\mathbf{r}_1$, $\mathbf{r}_2$ are the two-dimensional vectors. We set the electron charge equal to $-e$.

In a strong magnetic field the Bohr radii of the electrons $a_{\mathrm{B}}^{(1)} = \varepsilon_0\hbar^2/m_1e^2$, $a_{\mathrm{B}}^{(2)} = \varepsilon_0\hbar^2/m_2e^2$ can be much larger than the magnetic length $\ell_{\mathrm{B}} = (c\hbar/eB)^{1/2}$. In this case the Coulomb interaction can be taken into account as a perturbation. It is known that the quantum problem of a particle in a quantizing magnetic field has a large degeneracy (in the symmetric gauge with respect to the quantum number m, the z projection of the angular momentum). Therefore, the common formulation of the theory of perturbations should be based on a solution

of a secular equation. But such an approach is not an optimal method for the study of this problem. Here we use another approach, taken from the quantum Hall theory [16]. In this paper we use the method of [17], based on the projection of the Hamiltonian (1) into the subspace of the states of the pair of electrons in which the electrons in the layers 1 and 2 are frozen on the zero and the first Landau levels, respectively (this is exactly the same approach as is used in the theory of the quantum Hall effect). Then the kinetic energy operator for the electron in layer 1

$$H_1 = \frac{\Pi_{1x}^2 + \Pi_{1y}^2}{2m_1} = \hbar\omega_1\left(a_1^+ a_1 + \frac{1}{2}\right) \tag{5}$$

is projected onto $\bar{H}_1 = \hbar\omega_1/2$ (here and further the bar symbols indicate the projected operators) which is constant. We will omit this constant in the further consideration. In (5) $\omega_1 = eB/m_1c$ is the cyclotron frequency for the electron in layer 1, $\Pi_{1x} \equiv p_{1x} + \frac{eB}{2c}y_1$, $\Pi_{1y} \equiv p_{1y} - \frac{eB}{2c}x_1$ are the electron kinematic momentum components, $a_1^+ = \frac{\ell_B}{\hbar\sqrt{2}}(\Pi_{1x} - \mathrm{i}\Pi_{1y})$, $a_1 = \frac{\ell_B}{\hbar\sqrt{2}}(\Pi_{1x} + \mathrm{i}\Pi_{1y})$ are the creation and annihilation operators for the electron in layer 1. It follows from the commutation relation for Π_i^1 ($[\Pi_{1x}, \Pi_{1y}] = \mathrm{i}\hbar^2/\ell_B^2$) that $[a, a^+] = 1$, as it should. Analogously, for the electron in layer 2 one can find

$$H_2 = \frac{\Pi_{2x}^2 + \Pi_{2y}^2}{2m_2} = \hbar\omega_2\left(a_2^+ a_2 + \frac{1}{2}\right) \tag{6}$$

and $\bar{H}_2 = 3\hbar\omega_2/2$.

To project out the $V_C(|\mathbf{r}_1 - \mathbf{r}_2|)$ operator it is convenient to rewrite it in a Fourier-representation form (we follow the procedure of [17]):

$$V_C = \frac{e^2}{2\pi\varepsilon_0}\int \mathrm{d}^2k\,\frac{\exp(-k|d|)}{|k|}\exp(\mathrm{i}k_x(x_1 - x_2) + \mathrm{i}k_y(y_1 - y_2)), \tag{7}$$

where $|k| = \sqrt{k_x^2 + k_y^2}$.

The coordinates of the electron in layer 1 can be presented as

$$x_1 = X_1 - \frac{\ell_B^2}{\hbar}\Pi_{1y}, \qquad y_1 = Y_1 + \frac{\ell_B^2}{\hbar}\Pi_{1x}, \tag{8}$$

where X_1, Y_1 are the coordinates of the centre of its orbit in the magnetic field. The operators X_1 and Y_1 satisfy the following commutation relations: $[X_1, Y_1] = -\mathrm{i}\ell_B^2$. Beside that, X_1 and Y_1 commute with the momenta components Π_{1x} and Π_{1y}. Analogously, for the electron in layer 2 we have

$$x_2 = X_2 + \frac{\ell_B^2}{\hbar}\Pi_{2y}, \qquad y_2 = Y_2 - \frac{\ell_B^2}{\hbar}\Pi_{2x}, \tag{9}$$

where $[X_2, Y_2] = \mathrm{i}\ell_B^2$. Now the projection of the operator V_C is reduced to the independent projection of two commuting operators U_1 and U_2:

$$U_1 \equiv \exp\left(-\mathrm{i}k_x\frac{\ell_B^2}{\hbar}\Pi_{1y} + \mathrm{i}k_y\frac{\ell_B^2}{\hbar}\Pi_{1x}\right) = \exp\left[\frac{\ell_B}{\sqrt{2}}(ka_1^+ - \bar{k}a_1)\right], \tag{10}$$

$$U_2 \equiv \exp\left(\mathrm{i}k_x\frac{\ell_B^2}{\hbar}\Pi_{2y} - \mathrm{i}k_y\frac{\ell_B^2}{\hbar}\Pi_{2x}\right) = \exp\left[\frac{\ell_B}{\sqrt{2}}(\bar{k}a_2^+ - ka_2)\right], \tag{11}$$

where the notation $k \equiv k_x + ik_y$ is used. The projection of these operators can be easily performed:

$$\begin{aligned} \bar{U_1} &= \langle 0|U_1|0\rangle = \exp\left(-\frac{|k|^2\ell_B^2}{4}\right), \\ \bar{U_2} &= \langle 1|U_2|1\rangle = \exp\left(-\frac{|k|^2\ell_B^2}{4}\right)\left[1-\frac{|k|^2\ell_B^2}{2}\right]. \end{aligned} \tag{12}$$

Substituting equation (12) into (7) we arrive at the following expression for the operator $\bar{V}_C$:

$$\bar{V}_C = \frac{e^2}{2\pi\varepsilon_0}\int d^2k\,\frac{e^{-|k|d}}{|k|}e^{-|k|^2\ell_B^2/2}\left[1-\frac{|k|^2\ell_B^2}{2}\right]e^{ik_x(X_1-X_2)+ik_y(Y_1-Y_2)}. \tag{13}$$

Since the operators X_1-X_2 and Y_1-Y_2 commute with each other and with the Hamiltonian (1), these operators are the integrals of motion. The appearance of the integrals of motion in the problem considered is not accidental. The point is that Hamiltonian (1) and the Hamiltonian of the electron–hole pair coincide with each other up to the sign of the Coulomb interaction. In the last (electron–hole) case the integral of motion is the momentum of the pair **P** [18], which is the eigenvalue of the operator

$$\vec{\mathcal{P}} = \left(-i\hbar\frac{\partial}{\partial\mathbf{r}_1}+\frac{e}{c}\mathbf{A}_1\right)+\left(-i\hbar\frac{\partial}{\partial\mathbf{r}_2}-\frac{e}{c}\mathbf{A}_2\right)-\frac{e}{c}[\mathbf{B}\times(\mathbf{r}_1-\mathbf{r}_2)]. \tag{14}$$

In our problem the momentum of the pair **P** is also an integral of motion. Comparing equation (14) with (8) and (9) we find the relation between the components of the operator of the momentum $\vec{\mathcal{P}}$ and the operators $X_1 - X_2$ and $Y_1 - Y_2$:

$$\begin{aligned} &\mathcal{P}_x = \frac{\hbar}{\ell_B^2}(Y_2-Y_1) \qquad \text{and} \qquad \mathcal{P}_y = -\frac{\hbar}{\ell_B^2}(X_2-X_1) \\ &\text{or} \qquad \vec{\mathcal{P}} = \frac{\hbar}{\ell_B^2}(\mathbf{R}_2-\mathbf{R}_1)\times e_z, \end{aligned} \tag{15}$$

where e_z is the unit vector in the z direction. Taking equation (15) into account we rewrite the energy of the electron pair as the function of its momentum **P**:

$$\Delta E_{01}(\mathbf{P}) = \frac{e^2}{2\pi\varepsilon_0}\int d^2k\,\frac{e^{-|k|d}}{|k|}\exp\left(\frac{ik_x\ell_B^2}{\hbar}P_y-\frac{ik_y\ell_B^2}{\hbar}P_x\right)\exp\left(-\frac{|k|^2\ell_B^2}{2}\right)\left(1-\frac{|k|^2\ell_B^2}{2}\right). \tag{16}$$

Prior to analysing equation (16) we note the following.

(1) Using the method presented here one can also find the dependence of the energy of the pair on its momentum **P** in a general case, when the electron in layer 1 is 'frozen' on the n_1th Landau level and the electron in layer 2 is frozen on the n_2th level. The final result is

$$\begin{aligned} \Delta E_{n_1n_2}(\mathbf{P}) = \frac{e^2}{2\pi\varepsilon_0}\int d^2k\,\frac{e^{-|k|d}}{|k|}\exp\left(\frac{ik_x\ell_B^2}{\hbar}P_y-\frac{ik_y\ell_B^2}{\hbar}P_x\right) \\ \times\exp\left(-\frac{|k|^2\ell_B^2}{2}\right)L_{n_1}\left(\frac{|k|^2\ell_B^2}{2}\right)L_{n_2}\left(\frac{|k|^2\ell_B^2}{2}\right), \end{aligned} \tag{17}$$

where $L_n(x)$ are the Laguerre polynomials.

(2) The variant of the theory of perturbations used here allows us to solve the problem also in the case where the condition $a_B \gg \ell_B$ is satisfied only for the one particle of the pair, while the Bohr radius for the other particle can be of order ℓ_B. Such a situation may occur if the effective masses of the carriers differ considerably from each other ($m_1 \ll m_2$); see [15].

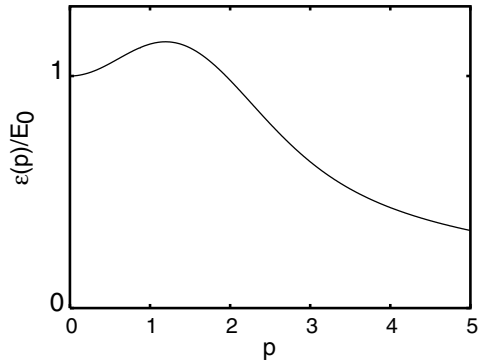

Figure 1. The dependence of the energy of the electron pair on its momentum.

Returning to the analysis of the result (16), at the first step we consider the case where the interlayer distance is equal to zero. In this case the analytical expression for the energy of the pair as a function of the momentum $\mathbf{P}$ can be found. At $d = 0$ the integral in the r.h.s. of equation (16) is calculated analytically. The result is

$$\Delta E_{01}(p) = \frac{e^2}{\varepsilon_0 \ell_B}\sqrt{\frac{\pi}{2}}\frac{1}{2}\left[\left(1-\frac{p^2}{2}\right)I_0\left(\frac{p^2}{4}\right) - \frac{p^2}{2}I_1\left(\frac{p^2}{4}\right)\right]e^{-p^2/4}, \tag{18}$$

where p is the modulus of the dimensionless momentum $\mathbf{p} = \mathbf{P}\ell_B/\hbar$ of the pair, $I_0(x)$ and $I_1(x)$ are the modified Bessel functions of zeroth and first order, respectively. Using the asymptotic expressions for $I_0(x)$ and $I_1(x)$ one can find from equation (18) the dependence $\Delta E_{01}(p) \equiv \epsilon(p)$ at small and large p.

(1) At $p \ll 1$,

$$\epsilon(p) \cong E_0 + \frac{\hbar^2 p^2}{2M_* \ell_B^2}, \tag{19}$$

where the energy E_0 and the effective mass of the pair M_* read as

$$E_0 = \frac{e^2}{2\varepsilon_0 \ell_B}\sqrt{\frac{\pi}{2}} \qquad \text{and} \qquad M_* = \left(\frac{2}{\pi}\right)^{1/2}\frac{4\varepsilon_0\hbar^2}{e^2\ell_B}. \tag{20}$$

One should note that in the approximation used the effective mass M_* is determined only by the interaction between electrons. The bare masses m_1 and m_2 do not enter into the expression for M_*. The bare masses determine only the Larmor frequencies ω_1 and ω_2.

(2) At $p \gg 1$,

$$\epsilon(p) \simeq \frac{4}{\sqrt{2\pi}}\frac{E_0}{p}. \tag{21}$$

As follows from equations (20) and (21), the energy of the electron pair as a function of the momentum p increases at small momenta and decreases at large p. Numerical estimates show that the function $\epsilon(p)$ reaches its maximum $\epsilon_m = 1.148E_0$ at $p = p_m = 1.194$. The dependence $\epsilon(p)$ is shown in figure 1. At $d \neq 0$ the integral in equation (16) cannot be evaluated analytically. Nevertheless, one can show (see the appendix) that the maximum of the function $\Delta E_{01}(p)$ survives at $d \neq 0$, but the energy barrier separating the metastable bound state of the pair from the ground state becomes smaller with increase of the ratio d/ℓ_B. At $d/\ell_B \gtrsim 0.8$ the energy barrier vanishes and the energy of the pair becomes a monotonically decreasing function of p.

To clarify the origin of the minimum and the maximum in the energy spectrum $\Delta E_{01}(p)$ it is instructive to present the expression (16) in terms of the wavefunction of the pair. Replacing the factor $\mathrm{e}^{-|k|d}/|k|$ in equation (16) with the integral

$$\frac{\mathrm{e}^{-|k|d}}{|k|} = \frac{1}{2\pi}\int \frac{\mathrm{e}^{-\mathrm{i}\mathbf{k}\cdot\mathbf{r}}}{\sqrt{r^2+d^2}}\,\mathrm{d}^2 r \tag{22}$$

and integrating the result over $\mathbf{k}$ one can rewrite the energy (16) in the form

$$\Delta E_{01}(p) = \frac{e^2}{\varepsilon_0}\int \frac{|\Psi(\mathbf{r}-\mathbf{r}_p)|^2}{\sqrt{r^2+d^2}}\,\mathrm{d}^2 r, \tag{23}$$

where

$$\mathbf{r}_p = \mathbf{B}\times\mathbf{p}\frac{\ell_\mathrm{B}}{B}, \tag{24}$$

$$|\Psi(\mathbf{r})|^2 \equiv \int \frac{\mathrm{d}^2 k}{(2\pi)^2}\,\mathrm{e}^{-\mathrm{i}\mathbf{k}\cdot\mathbf{r}}\mathrm{e}^{-|k|^2\ell_\mathrm{B}^2/2}\left(1-\frac{|k|^2\ell_\mathrm{B}^2}{2}\right) = \frac{1}{4\pi}\exp\left(-\frac{r^2}{2\ell_\mathrm{B}^2}\right)\frac{r^2}{\ell_\mathrm{B}^4}. \tag{25}$$

Equation (23) yields the correction to the energy of the electron pair due to the Coulomb interaction between electrons computed in the first order of the perturbation theory. Since the factor $e^2/\varepsilon_0\sqrt{r^2+d^2}$ is the energy of the Coulomb interaction between the electrons in the pair, the quantity $|\Psi(\mathbf{r}-\mathbf{r}_p)|^2$ is the squared modulus of the wavefunction of the pair with the momentum $\mathbf{p}$ (and, therefore, such notation is used for the quantity (25)).

The wavefunction of the pair can be found as a superposition of the products of the one-particle electron wavefunctions Ψ_1^0 in layer 1 (on the zero Landau level) and the wavefunctions Ψ_2^1 in layer 2 (on the first Landau level). In the Landau gauge $\mathbf{A} = (0, Bx, 0)$,

$$\Psi_1^0(k_1) = C_1(k_1)\mathrm{e}^{\mathrm{i}k_1 y_1}\exp\left[-\frac{(x_1+k_1\ell_\mathrm{B}^2)^2}{2\ell_\mathrm{B}^2}\right] \tag{26}$$

$$\Psi_2^1(k_2) = C_2(k_2)\mathrm{e}^{\mathrm{i}k_2 y_2}\exp\left[-\frac{(x_2-k_2\ell_\mathrm{B}^2)^2}{2\ell_\mathrm{B}^2}\right]\frac{x_2-k_2\ell_\mathrm{B}^2}{\ell_\mathrm{B}}. \tag{27}$$

One should note that the terms $k_1\ell_\mathrm{B}^2$ and $k_2\ell_\mathrm{B}^2$ in the exponents in equations (26) and (27) have opposite signs. This is connected with the magnetic fields in layers 1 and 2 being antiparallel to each other. The functions (26) and (27) coincide with the wavefunctions of an electron and a hole in the electron–hole pair in a *uniform* magnetic field.

Let us assume that the coefficients C_1 and C_2 do not depend on k_1 and k_2. Then the wavefunction of the electron pair with the total momentum $\mathbf{p}$, directed along the y axis, has the form

$$\Psi(\mathbf{r}-\mathbf{r}_p) = C\sum_q \Psi_1^0\left(\frac{p}{2\ell_\mathrm{B}}+q\right)\Psi_2^1\left(\frac{p}{2\ell_\mathrm{B}}-q\right). \tag{28}$$

Computing the sum over q in equation (28) and normalizing the function $\Psi(\mathbf{r}-\mathbf{r}_p)$ to unity, we find

$$\begin{aligned}\Psi(\mathbf{r}-\mathbf{r}_p) = {}& \frac{1}{\sqrt{\pi}}\frac{1}{\ell_\mathrm{B}}\exp\left(\frac{\mathrm{i}p(y_1+y_2)}{2\ell_\mathrm{B}}\right)\exp\left(-\mathrm{i}\frac{(x_1+x_2)(y_1-y_2)}{2\ell_\mathrm{B}^2}\right)\\ &\times\exp\left[-\frac{(x_1-x_2+p\ell_\mathrm{B})^2+(y_1-y_2)^2}{4\ell_\mathrm{B}^2}\right]\left(\frac{x_1-x_2+p\ell_\mathrm{B}}{2\ell_\mathrm{B}}-\mathrm{i}\frac{y_1-y_2}{2\ell_\mathrm{B}}\right).\end{aligned} \tag{29}$$

The squared modulus of the function (29) coincides with expression (25). This justifies the assumption that the coefficients C_1 and C_2 do not depend on k_1 and k_2. One can easily verify

that the function (29) is the eigenfunction of the momentum operator (14), taken in the Landau gauge. Therefore the energy of the pair (23) is a function of the conservative quantity p.

The dependence $\Delta E_{01}(p)$ at $d \ll \ell_B$ can be written as

$$\Delta E_{01}(p) = \frac{e^2}{\varepsilon_0} \int \frac{|\Psi(\mathbf{r})|^2}{|\mathbf{r} + \mathbf{r}_p|} \, \mathrm{d}^2 r. \tag{30}$$

One can easily see that the function $|\Psi(\mathbf{r})|^2$ has a maximum at $r = \sqrt{2}\ell_B$. At the same time, the maximum of the function $1/|\mathbf{r} + \mathbf{r}_p|$ is reached at $r = r_p$. At small momenta ($p\ell_B/\hbar \lesssim 1$), these two maxima become closer to each other with increase of p. In the opposite case of large momenta ($p\ell_B/\hbar \gtrsim 1$), the two maxima become more separated with increasing momenta. As a result, at small p the overlapping of two functions under integral (30) increases with increase of p and at such momenta E_{01} is an increasing function of p. At large p the overlapping decreases and E_{01} turns into a decreasing function of p. The key point is the nonmonotonic behaviour of the function $|\Psi(r)|^2$. Note that in the case of two electrons belonging to the zero Landau level, the function $|\Psi(r)|^2 \propto \exp(-r^2/2\ell_B^2)$ is a monotonic function of r and $\Delta E_{00}(p)$ is also a monotonic (decreasing) function of the momentum of the pair p.

3. Discussion

Let us discuss the consequences of the existence of the interval of p where the energy of the pair increases with increase of its momentum. The mean value of the squared distance between the electrons in the pair is

$$\langle r^2 \rangle = \int r^2 |\Psi(\mathbf{r} - \mathbf{r}_p)|^2 \, \mathrm{d}^2 r = \ell_B^2 (4 + p^2). \tag{31}$$

It follows from equation (31) that at any momentum p the pair has a finite size. Therefore, in antiparallel magnetic fields the momentum conservation results in the formation of a bound state of two electrons at any p. But such a state can be completely unstable. The reason is the interaction of electrons with the lattice that always takes place in real physical systems. Due to this interaction the pair can emit an acoustic or optical phonon and reduce its energy. In the interval of p where $\mathrm{d}E_{01}(p)/\mathrm{d}p < 0$, the emission of a phonon is a process without a threshold. The emission is accompanied with increase of the momentum of the pair and the average distance between electrons becomes larger. As a result, the distance between the electrons increases infinitely after many emissions and the bound state disappears. Another situation occurs in the region of p where $\mathrm{d}E_{01}(p)/\mathrm{d}p > 0$. In this case, as follows from the energy and momentum conservation laws, the emission of a phonon may take place under the condition that the momentum of the pair p is higher than a certain critical value p_c. Therefore, the pairs with the momenta $p < p_c$ do not decay at all (if one neglects exponentially small underbarrier tunnelling processes). The pairs with $p > p_c$ can emit phonons, but such processes lower the energy and momentum of the pair and the average distance between electrons after such events is decreased. Thus, the bound state of the pairs is stable with respect to the emission of phonons if the momenta of the pair satisfy the condition $\mathrm{d}E_{01}(p)/\mathrm{d}p > 0$,

The bound electron pairs are bosons and, therefore, in the system of such pairs one can expect a transition into a superfluid state if the density of the pair is quite large. Since the pairs are charged, this state should be a superconductive one. Strictly speaking, the problem of a transition of the electron pairs into the superfluid state in the system studied requires further analysis. Since the bound state of the pairs does not correspond to the true minimum of the energy, thermodynamic arguments cannot be used to establish the conditions for the existence of a superfluid state. To clarify this question, the probability of a transition from the state with

a given number of pairs with zero or small momenta should be found. We expect that, due to the presence of the barrier to pair decoupling in the two-particle problem, a barrier to the destruction of the coherent state will exist in the many-particle problem as well. Therefore, at temperature lower than the value of the barrier, the lifetime of the coherent state will be long.

In conclusion, we discussed briefly the question of the possibility of realizing the magnetic field configuration required. This question is not so simple from the experimental point of view. In fact, the fields should be of the order of 1–10 T. But we think this obstacle can be overcome. We propose two possible ways of solving this problem. First, the required (antiparallel) configuration of the magnetic fields can be realized using magnetized stripes of magnetically hard materials (such as Dy) deposited on the bilayer structure. In [19] such a method was used for designing periodic magnetic fields with $B_{\max} = 1$ T with the aim of studying the conducting properties of a two-dimensional electron gas in such fields. Another possible way to create an antiparallel magnetic field configuration is based on using antiferromagnetic systems in which the spins in each layer are ferromagnetically ordered while they are directed antiparallel in adjacent layers. For example, such properties are demonstrated by the layered manganites $(LaSr)_{n+1}Mn_nO_{3n+1}$, compounds which are being widely studied now (see, for instance, the review [20]).

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Appendix A. The dependence of the energy of the pair on its momentum at $d \neq 0$

At $d \neq 0$ equation (16) can be reduced to

$$\Delta E_{01}(p) = \frac{e^2}{\varepsilon_0 \ell_B} \int_0^\infty e^{-xd/\ell_B - x^2/2}(1 - x^2/2) J_0(xp)\, dx. \tag{A.1}$$

One can see that the function $\Delta E_{01}(p)$ depends on the dimensionless parameter $\delta = d/\ell_B$. At small p the Bessel function under the integral can be replaced by the series

$$J_0(xp) = 1 - \frac{x^2 p^2}{4} + \cdots. \tag{A.2}$$

Then

$$\Delta E_{01}(p) = \Delta E_{01}(0) + \frac{Ap^2}{2} + o(p^2). \tag{A.3}$$

It follows from equation (A.3) that there is an extremum of the function $\Delta E_{01}(p)$ at $p = 0$ ($\partial \Delta E_{01}(p)/\partial p = 0$): a minimum at $A > 0$ or a maximum at $A < 0$. The exact expression for A can be presented in terms of the parabolic cylinder function $D_\nu(z)$ [21]:

$$\begin{aligned} A(\delta) &= \frac{e^2}{2\varepsilon_0 \ell_B} \int_0^\infty dx\, x^2 \left(\frac{x^2}{2} - 1\right) \exp\left(-\delta x - \frac{x^2}{2}\right) \\ &= \frac{e^2}{\varepsilon_0 \ell_B} \exp\left(\frac{\delta^2}{4}\right)[6D_{-5}(\delta) - D_{-3}(\delta)]. \end{aligned} \tag{A.4}$$

Using the recurrence relation $D_{p+1}(0) + pD_{p-1}(0) = 0$ and the value of $D_{-1}(0) = \sqrt{\pi/2}$ [22] we find that $D_{-3}(0) = \sqrt{\pi/8}$ and $D_{-5}(0) = \sqrt{\pi/128}$. Therefore, the quantity

$$A(0) = \frac{e^2}{4\varepsilon_0 \ell_B} \sqrt{\frac{\pi}{2}}$$

is positive. This means that at $d = 0$ the function $\Delta E_{01}(0)$ has a minimum at $p = 0$, in accordance with the result (19).

The condition $A(\delta_c) = 0$, which is equivalent to

$$6D_{-5}(\delta_c) - D_{-3}(\delta_c) = 0 \tag{A.5}$$

determines the critical value δ_c: at $\delta > \delta_c$ the energy barrier disappears and the bound state becomes unstable. For physical reasons one can assume that $\delta_c \lesssim 1$. Using the asymptotic expression for $D_\nu(z)$ [22],

$$D_\nu(z) \simeq \frac{1}{\sqrt{2}} \exp\left[\frac{\nu}{2}\ln(-\nu) - \frac{\nu}{2} - \sqrt{-\nu}x\right], \tag{A.6}$$

valid for $|\nu| \gg z$, the condition (A.5) can be reduced to the form

$$\exp[-\delta_c(\sqrt{5} - \sqrt{3})] = \frac{5}{6e}\left(\frac{5}{3}\right)^{3/2}.$$

This yields

$$\delta_c = \frac{1 + \ln\left(\frac{18}{25}\sqrt{\frac{3}{5}}\right)}{\sqrt{5} - \sqrt{3}} \approx 0.83. \tag{A.7}$$

Note that the answer (A.7), found analytically, is very close to the value of $\delta_c (\approx 0.78)$ obtained by direct numerical evaluation of the integral (A.1).

Thus, as follows from our analysis, the metastable bound state of electrons in the systems considered can be realized at the interlayer distances $d < \ell_B \delta_c$.

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